DIFFUSION OF CHEMICALLY REACTIVE SPECIES IN A VISCOELASTIC FLOW OVER A SHRINKING SHEET IN THE PRESENCE OF A MAGNETIC FIELD

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ABSTRACT

This paper deals with the study of diffusion of reactive species undergoing first-order chemical reaction in a boundary layer flow of an incompressible homogeneous second order fluid over a linearly shrinking sheet in the presence of a transverse magnetic field. Using similarity variables, the boundary layer equations governing the flow and concentration field are reduced into a set of nonlinear ordinary differential equations. Exact solutions of the reduced equations are obtained for power-law surface concentration (PSC) as well as power-law wall mass flux (PMF) boundary conditions. The study reveals that the velocity is getting more closer towards wall for increasing magnetic parameter whereas it is going away from the wall for increasing viscoelastic parameter. It is also found that the diffusion of reactive species is considerably reduced with increasing values of Schmidt number, magnetic and reaction rate parameter whereas it is increased for enhanced values of viscoelastic parameter for both PSC and PMF cases. For PSC case, the concentration boundary layer thickness is enhanced with the increasing values of power-law index parameter. Negative concentration is observed in some cases which may not have real world applications.

Keywords: Shrinking sheet, Viscoelastic fluid, Boundary layer flow, MHD, Diffusion, Chemically reactive species, Exact Solution

1 INTRODUCTION

Viscoelastic fluid flow over a stretching sheet have numerous applications in several industrial manufacturing processes. Some of the typical applications of such flows are polymer sheet extrusion from a die, drawing of plastic films, glass fiber and paper production etc. Rajagopal et al. (1984) gave a mathematical formulation of viscoelastic fluid flow over a stretching sheet and obtained an approximate mathematical solution of it. Later, this problem was extended by Rajagopal et al. (1987) by introducing uniform free stream velocity in the problem formulation. Bujurke et al. (1987) presented the momentum and heat transfer phenomena in incompressible second order fluid over a stretching sheet with internal heat generation and viscous dissipation. Dandapat and Gupta (1989) studied the viscoelastic flow and heat transfer over a stretching sheet. Rollins and Vajravelu (1991) solved the heat transfer problem in a second order fluid over a continuous stretching surface. Heat transfer in the viscoelastic fluid over a stretching sheet conditions.
in different context was also studied by Lorence and Rao(1992), Bhattacharya et al. (1998), Abel et al. (2002, 2007) and others. Magnetic influences on these flows were studied by many researchers. Andersson (1992) investigated the flow problem of electrically conducting viscoelastic fluid past a flat and impermeable elastic sheet. Lawrence and Rao (1995) studied the non-uniqueness of the MHD flow of second order fluid past a stretching sheet. Chowdhury and Das (2010) studied the hydromagnetic flow and heat transfer of a viscoelastic fluid over a moving vertical surface. Heat transfer in a viscoelastic flow over a stretching sheet in the presence of magnetic field was studied by Char (1994), Khan et al. (2003), Datti et al. (2004), Cortell (2006), Abel and Mahesha (2008), Prasad et al. (2010) and others.

Again, the mass transfer in a viscoelastic boundary layer fluid flow have added a new arena of research because of its huge engineering applications in polymer technology, metallurgy and chemical industries. Prasad et al. (2003) studied the diffusion of chemically reactive species of a non-Newtonian fluid immersed in a porous medium over a stretching sheet. Recently, Cortell (2007a, 2007b) discussed the effects of magnetic field on the flow and mass transfer of a second grade fluid in porous medium over a stretching sheet with chemically reactive species and also explained the motion and mass transfer with chemically reactive species for two classes of viscoelastic fluid past a porous stretching sheet subjected to applied suction or blowing. The diffusion of first order chemical reaction on the viscoelastic fluid flow past an infinite vertical porous plate was studied by Damesh and Shannak (2010).

On the other hand, the flow over a shrinking sheet is a new field of research at present and few literature are available on this area of research now. Wang (1990) first studied a specific shrinking sheet problem. Recently, Miklavcic and Wang (2006) obtained the existence and uniqueness of the solution for steady viscous hydrodynamic flow over a shrinking sheet with mass suction. Hayat et al. (2007) derived both exact and series solution (using HAM) describing the magnetohydrodynamic boundary layer flow of a second grade fluid over a shrinking sheet. The problem of MHD viscous flow due to a shrinking sheet was solved by Sajid and Hayat (2009) using HAM. Fang and Zhang (2009) obtained a closed-form analytical solution for steady MHD flow over a porous shrinking sheet subjected to wall mass suction. Noor et al. (2010) found a solution in the form of an infinite series of MHD viscous flow over a shrinking sheet by applying Adomian decomposition method (ADM). Mahapatra and Nandy (2011) analyzed the unsteady flow and heat transfer in the neighbourhood of a stagnation-point over a shrinking sheet in the presence of time-dependent free stream. Midya (2012a) studied the magnetohydrodynamic viscous flow and heat transfer over a linearly shrinking porous sheet. Hayat et al. (2008) analyzed the MHD flow and mass transfer of an upper-convected Maxwell fluid past a porous shrinking sheet with chemical reaction using the homotopy analysis method (HAM). Effect of chemical reaction, heat and mass transfer on nonlinear boundary layer past a porous shrinking sheet in the presence of suction was studied numerically by Muhammin et al. (2010). Recently, Midya (2012b) found a closed form analytical solution for the distribution of reactant solute in a MHD boundary layer flow over a permeable shrinking sheet.

The aim of this work is to investigate analytically the diffusion of chemically reactive species on viscoelastic fluid flow over a linearly shrinking sheet in the presence of magnetic field. Both prescribed power-law surface concentration (PSC) and power-law wall mass flux (PMF) cases are considered as surface boundary conditions. Using the similarity solution technique, the governing partial differential equations are transformed into a set of nonlinear self-similar ordinary differential equations which are then solved analytically. Closed form exact solutions of the concentration equation are obtained in terms of Kummer’s function. The effects of various parameters on the concentration distribution are analyzed and are presented graphically.
2 FORMULATION OF THE PROBLEM

Consider the steady two-dimensional laminar flow of an incompressible second-order viscoelastic electrically conducting fluid over a shrinking surface. Magnetic induction \( B_0 \) is applied perpendicular to the shrinking surface. \( x \)-axis is chosen in the direction opposite the sheet motion and the \( y \)-axis is taken perpendicular to it. The shrinking sheet velocity is proportional to the distance i.e. \( u_w = -ax, \quad (a > 0) \). Using boundary layer approximation and neglecting the induced magnetic field (by assuming the magnetic Reynolds number \( R_m \) for the flow to be very small i.e. \( R_m << 1 \) [see Midya et al. (2003)], the equations for steady two-dimensional MHD flow and the reactive concentration equation can be written in usual notation as

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - k_0 \left\{ \frac{u}{x} \frac{\partial^3 u}{\partial y^3} + \frac{\partial^3 u}{\partial x \partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right\} - \frac{\sigma B_0^2}{\rho} u \quad (2)
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1 (C - C_\infty) \quad (3)
\]

where \( u \) and \( v \) are the components of velocity respectively in the \( x \) and \( y \) directions, \( C \) is the concentration, \( \rho \) is the fluid density (assumed constant), \( \sigma \) is the electrical conductivity of the fluid, \( \nu \) is the coefficient of fluid viscosity, \( D \) is the mass diffusion coefficient, \( k_1 \) is the chemical reaction coefficient.

The boundary conditions for the velocity components and concentration are given by

\[
u = -ax, \quad v = 0, \quad C = C_w \quad \text{at} \quad y = 0, \quad (4)
\]

and

\[
u \to 0, \quad C \to C_\infty \quad \text{as} \quad y \to \infty. \quad (5)
\]

where \( C_w \) is the wall concentration, \( C_\infty \) is the concentration far from the sheet.

3 SOLUTION OF THE PROBLEM

The stream function and the similarity variable can be posited in the form

\[
\Psi (x, y) = \sqrt{ax} f(\eta), \quad \eta = y \sqrt{\frac{a}{\nu}} \quad (6)
\]

We, therefore, have

\[
u = ax f'(\eta), \quad v = -\sqrt{ax} f(\eta). \quad (7)
\]

The similarity equation now becomes

\[
\frac{f'''}{f'} - f'' = f''' - k \left\{ 2 f' f''' - f''^2 - f f'' \right\} - M^2 f', \quad (8)
\]

where \( M = \sqrt{\frac{\sigma B_0^2}{\alpha \nu}} \) and \( k = \frac{k_{\text{ch}}}{\nu} \) are the magnetic interaction and visco-elastic parameters respectively.
The boundary conditions are

\[ f(0) = 0, \quad f'(0) = -1, \quad \text{and} \quad f'(\infty) = 0. \]  \hspace{1cm} (9)

It can be verified that

\[ f(\eta) = \frac{1}{\alpha}(e^{-\alpha \eta} - 1), \quad \alpha = \sqrt{\frac{M^2 - 1}{1 + k}}, \]  \hspace{1cm} (10)

is a solution of the equation (8) with associated boundary conditions. It is seen that this solution is valid for \( M > 1 \).

Now we use the above solution [Eq. (10)] for momentum transport equation to analyse the diffusion of reactive species in the flow field. In the present work, prescribed power-law surface concentration (PSC) as well as power-law wall mass flux (PMF) cases are taken as the surface boundary conditions.

### 3.1 POWER-LAW SURFACE CONCENTRATION (PSC) CASE:

In the power-law surface concentration (PSC) case, we take the boundary conditions as

\[ C_w = C_\infty + Ax^p \quad \text{at} \quad y = 0 \]  \hspace{1cm} (11)

and

\[ C = C_\infty \quad \text{at} \quad y \to \infty. \]  \hspace{1cm} (12)

Defining the non-dimensional concentration \( \theta(\eta) \), Schmidt number \( Sc \) and reaction rate parameter \( \gamma \) as

\[ \theta(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad Sc = \frac{\nu}{D}, \quad \gamma = \frac{k_1}{a} \]  \hspace{1cm} (13)

and using Eqs.(6) and (7), one can obtain from Eq.(3),

\[ \theta'' + Scf\theta' - Sc(\gamma + pf')\theta = 0. \]  \hspace{1cm} (14)

The boundary conditions then become

\[ \theta(0) = 1, \quad \text{and} \quad \theta(\infty) = 0. \]  \hspace{1cm} (15)

Substituting the solution for the momentum transport equation in Eq.(14), we get

\[ \theta'' + \frac{Sc}{\alpha}(e^{-\alpha \eta} - 1)\theta' - Sc(\gamma - p e^{-\alpha \eta})\theta = 0. \]  \hspace{1cm} (16)

Now, let us introduce a new variable \( \xi = \frac{Sc}{\alpha^2} e^{-\alpha \eta} \) so that the above equation transforms to

\[ \xi \frac{d^2 \theta}{d\xi^2} + \left(1 + \frac{Sc}{\alpha^2} - \xi\right) \frac{d\theta}{d\xi} - \left(\frac{Sc\gamma}{\alpha^2 \xi} - p\right) \theta = 0. \]  \hspace{1cm} (17)

The boundary conditions are
\[ \theta \left( \frac{Sc}{\alpha^2} \right) = 1, \quad \text{and} \quad \theta(0) = 0 \] (18)

Now, transforming the above equation (17) into confluent hypergeometric equation, we can obtain the solution [see Abramowitz and Stegun (1972)] given by
\[ \theta(\xi) = \left( \frac{\alpha^2 \xi}{Sc} \right)^{\beta} \Phi(\beta - p, 1 + b_0, \xi) / \Phi(\beta - p, 1 + b_0, Sc/\alpha^2), \]
where \( \beta = (b_0 - a_0)/2, a_0 = Sc/\alpha^2, b_0 = \sqrt{a_0^2 + 4a_0\gamma} \) and \( \Phi(a, b, x) \) is the confluent hypergeometric function of the first kind or Kummer function.

Therefore,
\[ \theta(\eta) = e^{-\alpha\beta\eta} \Phi(\beta - p, 1 + b_0, Sc e^{-\alpha\eta}/\alpha^2) / \Phi(\beta - p, 1 + b_0, Sc/\alpha^2). \]

### 3.2 POWER-LAW MASS FLUX (PMF) CASE:

Now, we consider power-law wall mass flux (PMF) case. Here the boundary conditions become
\[ -D \frac{\partial C}{\partial y} = m_w = Ex^n \quad \text{at} \quad y \rightarrow \infty. \]
and
\[ C = C_\infty \quad \text{at} \quad y = \infty, \] (19)
where \( E \) is a constant, \( n \) the concentration flux power index, \( m_w \) is the wall concentration flux.

Defining the non-dimensional concentration by
\[ \omega(\eta) = \frac{C - C_\infty}{Ex^n D\sqrt{\gamma}} \] (20)
and using (6) and (7), we get from Eq.(3)
\[ \omega'' + Scf \omega' - Sc(\gamma + n f') \omega = 0. \] (21)

The boundary conditions then become
\[ \omega'(0) = -1, \quad \text{and} \quad \omega(\infty) = 0. \] (22)

Substituting the solution for the momentum transport in Eq.(21), we get
\[ \omega'' + \frac{Sc}{\alpha} (e^{-\alpha\eta} - 1) \omega' - Sc(\gamma - ne^{-\alpha\eta}) \omega = 0. \] (23)

Introduction of the new variable \( \xi = \frac{Sc}{\alpha^2} e^{-\alpha\eta} \) transforms it to
\[ \xi \frac{d^2 \omega}{d\xi^2} + \left( 1 + \frac{Sc}{\alpha^2} - \xi \right) \frac{d\omega}{d\xi} - \left( \frac{Sc\gamma}{\alpha^2 \xi} - n \right) \omega = 0. \] (24)

and the corresponding boundary conditions become
\[ \omega'(Sc/\alpha^2) = \alpha/Sc, \quad \text{and} \quad \omega(0) = 0. \] (25)

Now, the solution of above equation (24) with the boundary conditions is
\[
\omega(\xi) = \frac{\alpha (1 + b_0) \left( \frac{\alpha^2 \xi}{Sc} \right)^\beta \Phi (\beta - n, 1 + b_0, \xi)}{\alpha^2 \beta (1 + b_0) \Phi (\beta - n, 1 + b_0, \frac{Sc}{\alpha^2}) + Sc (\beta - n) \Phi (1 + \beta - n, 2 + b_0, \frac{Sc}{\alpha^2})}. \]

The solution then becomes
\[
\omega(\eta) = \frac{\alpha (1 + b_0) e^{-a\beta q} \Phi (\beta - n, 1 + b_0, \frac{Sc}{\alpha^2} e^{-a\eta})}{\alpha^2 \beta (1 + b_0) \Phi (\beta - n, 1 + b_0, \frac{Sc}{\alpha^2}) + Sc (\beta - n) \Phi (1 + \beta - n, 2 + b_0, \frac{Sc}{\alpha^2})}. \] (26)
in terms of \( \eta \).

4 DISCUSSIONS

In this section some examples will be calculated and discussed for certain values of the controlling parameters.

Figure 1 displays the velocity distributions for different magnetic parameter \( M \) with fixed viscoelastic parameter \( k = 0.2 \). It is seen that the velocity is going closer to the wall and boundary layer thickness becomes thinner for larger magnetic field parameter. This is due to the fact that the increase in \( M \) results the increase in Lorentz force which in turn produce more resistance to the velocity field.

The velocity curves for various values of viscoelastic parameter \( k \) have been plotted in Figure 2 for fixed magnetic parameter \( M = 2 \). The figure reflects that velocity profiles are going away from the wall for increasing \( k \) and the boundary layer thickness becomes thicker.

We now discuss the reactive species distribution in the flow field for prescribed power-law surface concentration(PSC) case followed by power-law wall mass flux(PMF) case.

Figure 3 demonstrate the effects of Schmidt number \( Sc \) on the concentration profile. It is seen that concentration is decreased for enhanced values of \( Sc \) for \( M = 2, p = 0, k = 0.1, \gamma = 0.2 \). Increasing in Schmidt number means decrease in diffusion coefficient. Therefore, decrease in diffusion coefficient leads to decrease the concentration in the flow field.

The effect of reaction rate parameter on the diffusion of chemically reactive solute is presented in Figure 4 for fixed values of \( k=0.1, Sc=0.8, M=2 \) and \( p=1 \). We see that concentration boundary layer is decreased for enhanced values of reaction rate parameter \( \gamma \). So in the case of the distribution of reactive solute, the reaction rate is acting as a decelerating agent and it thins the solute boundary layer thickness formed in the neighbourhood of the sheet.

Now, we shall draw our attention to the effects of solute distribution when the initial distribution of solute is varied over the sheet. The concentration profiles for different values of power-law exponent \( p \) with \( M=2, \gamma = 0.1, Sc = 0.5, k = 0.1 \) are plotted in Figure 5. It is observed from the figure that the rate of solute transfer is increased with the increase of concentration at the shrinking sheet.

Figure 6 is the graphical representation of concentration profiles for various values of M with fixed \( k=0.1, \) \( Sc=1, \gamma = 0.1, p=1 \) for PSC case. The figure reveals that the value of concentration at a particular \( \eta \) is reduced with increasing values of magnetic field parameter M. This implies that the magnetic force acts to distract the diffusion of reactive solute in a shrinking sheet.

Figure 7 displays the concentration boundary layer thickness for different values of the viscoelastic parameter \( k \) in PSC case. Other parameters are \( M = 2, Sc = 0.6, \gamma = 0.05, p = 2. \) It is observed that concentration boundary layer thickness is increased for increasing \( k. \)

Next, we concentrate on the PMF cases which have been presented from Figures 8-12. First we discuss the effect of Sc on the diffusion of the reactive solute and it has been displayed in Figure 8 for fixed values of \( M = 2, k = 0.1, \gamma = 0.2, n = 0. \) The figure shows that concentration boundary layer thickness decreases for increasing Sc. Actually, Schmidt number is inversely proportional to the diffusion coefficient. Hence it is decreasing for increasing Sc. It is evident from this figure that the concentration takes it’s limiting value \( C_\infty \) for a large distance from the sheet.

The effects of reaction rate parameter \( \gamma \) on the concentration in PMF case are presented in Figure 9. Here also, the reaction rate parameter is acting as a decelerating agent and as a result the solute boundary layer becomes thinner.

The effects of mass flux power index on the concentration are shown in Figure 10 for \( M = 2, Sc = 0.6, k = 0.1, \gamma = 0.1. \) In this case, negative concentration is noticed for \( n=1 \) and \( 2. \) The negative concentration values may not have real world applications.

Figure 11 illustrate the influence of magnetic parameter M on the concentration for \( Sc = 1, k = 0.1, \gamma = 0.2 \) and \( n = 0. \) The figure reveals that the diffusion of reactive solute as well as concentration boundary layer thickness is decreased with the increase in M. The destructive role of M in the diffusion of reactive solute is noticed in the PMF case too.

The concentration profiles for various values of viscoelastic parameter \( k \) are depicted in Figure 12 for \( M = 2, Sc = 0.6, \gamma = 0.05, n = 0. \) It is seen that rate of solute transfer is increased for enhanced values of viscoelastic parameter \( k \) in PMF case.

Figure 1: Velocity profiles \( f'(\eta) \) for various values of magnetic parameter \( M (M = 1.5,2,3) \) and \( k = 0.2. \)

Figure 2: Velocity profiles $f'(\eta)$ for various values of visco-elastic parameter $k$ ($k = 0.1, 0.5, 0.8$) and $M = 2$.

Figure 3: Variation of concentration for several values of $Sc$ ($Sc = 0.5, 1.0, 1.5$), $M = 2$, $k = 0.1$, $\gamma = 0.2$ and $p = 0$ in PSC case.

Figure 4: The concentration profiles for $\gamma$ ($= 0.05, 0.1, 0.2$), $M = 2$, $k = 0.1$, $Sc = 0.8$ and $p = 1$ in PSC case.
Figure 5: The concentration distribution for several values of $p$ ($p = 0, 2, 4$) with $M = 2$, $\gamma = 0.1$, $k = 0.1$, $Sc = 0.5$ (for PSC case).

Figure 6: The concentration profiles for several values of $M$ ($M = 1.5, 2, 3$), $\gamma = 0.1$, $k = 0.1$, $Sc = 1$, and $p = 1$ for PSC case.

Figure 7: Variation of concentration for several values of $k$ ($k = 0.1, 0.5, 0.8$), $M = 2$, $Sc = 0.6$, $\gamma = 0.05$ and $p = 2$ in PSC case.
Figure 8: Variation of concentration for several values of $Sc$ ($Sc = 0.5, 1.0, 1.5$), $M = 2$, $k = 0.1$, $\gamma = 0.2$ and $n = 0$ for PMF case.

Figure 9: The concentration profiles for $\gamma$ (= 0.05, 0.1, 0.2), $M = 3$, $k = 0.1$, $Sc = 0.6$ and $n = 0$ for PMF case.

Figure 10: The concentration distribution for several values of $n$ ($n = 0, 1, 2$) with $M = 2$, $\gamma = 0.1$, $k = 0.1$, $Sc = 0.6$ in PMF case.

Chemically reactive species distribution in a viscoelastic flow over a shrinking surface in the presence of magnetic field is investigated analytically for power-law surface concentration (PSC) and power-law mass flux (PMF) cases. Using similarity variables, the boundary layer equations governing the flow and concentration field are reduced into a set of nonlinear ordinary differential equations which are then solved. It is noticed that the velocity is getting more closer towards wall for increasing $M$ whereas it is going away from the wall for increasing $k$. It is also found that the concentration boundary layer thickness is enhanced with the increasing values of power-law index parameter and viscoelastic parameter, but it is decreased with enhanced values of Schmidt number, magnetic and reaction rate parameters in PSC case. For PMF case, the rate of solute transport is considerably reduced with increasing values of Schmidt number, magnetic and reaction rate parameter whereas it is increased for enhanced values of viscoelastic parameter. Negative nondimensional concentration is observed in PMF case for power-law index $n = 1$ and 2 when $M = 2, Sc = 0.6, k = 0.1, \gamma = 0.2$.

**NOMENCLATURE:**

- $a$: proportionality constant of the velocity of the sheet
- $u$: velocity component along the sheet

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$\nu$ velocity component normal to the sheet
$x$ distance along the sheet
$y$ distance normal to the sheet
$D$ mass diffusion coefficient
$k_1$ chemical reaction coefficient
$C$ concentration of the fluid
$C_w$ concentration of the wall of the surface
$C_\infty$ free-stream concentration
$f$ non-dimensional stream function
$p$ wall concentration power index
$n$ wall mass flux power index
$Sc$ Schmidt number
$m_w$ wall mass flux
$M$ magnetic parameter

Greek symbols:
$\xi$ transformation parameter
$\eta$ similarity variable
$\mu$ dynamic viscosity
$\sigma$ electrical conductivity of the fluid
$\nu$ kinematic viscosity
$\psi$ stream function
$\rho$ density of the fluid
$\gamma$ reaction rate parameter
$\theta$ non-dimensional concentration
$\omega$ non-dimensional concentration

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