LIE GROUP ANALYSIS OF HYDROMAGINATIC FLOW AND HEAT TRANSFER BY NON-DARCY NATURAL CONVECTION OVER A SURFACE STRETCHING IN POROUS MEDIUM WITH RADIATION EFFECT

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ABSTRACT

Lie symmetry group method is applied to analyze problem of hydromagnatic flow and heat transfer by natural convection over a vertical porous surface saturated porous medium in the presence magnetic field, non-Darcy and thermal radiation effects. The surface of the plate is maintained at a uniform heat flux and is a permeable to allow for possible fluid wall suction or blowing in the *z*-direction. Also, the flow is subjected to a uniform magnetic field in *z*-direction. By using Lie group method, we first find the general symmetries of the partial differential equations of the governing system. We then reduce the equations to an ordinary, differential system via similarity transformations. Finally, we solve numerically the resulting ordinary differential system using a fourth-order Runge-Kutta scheme. The numerical results obtained for various values of the Prandtl number, Hartmann number, Darcy number, porous-medium inertia coefficient and radiation parameter, reveal the influence of the parameters on the flow, heat transfer behavior.

Keywords: Lie group method, hydromagnatic flow, porous medium, surface stretching, non-Darcy, thermal radiation.

1 INTRODUCTION

Natural convection in a fluid-saturated porous medium is of fundamental importance in many industrial and natural problems. Few examples of the heat transfer by natural convection can be found in geophysics and energy related engineering problems such as natural circulation in geothermal reservoirs, aquifers, porous insulations, solar power collectors, spreading of pollutants etc. Natural convection occurs due to the spatial variations in density, which is caused by the non-uniform distribution of temperature or/and concentration of a dissolved substance. There are many studies (Nield and Bejan 1999; Vafai 2000; Pop and Ingham 2001; Pop and Ingham 2002; Bejan and Kraus 2003) in which natural convection caused by immersing a hot surface in a fluid-saturated porous medium at constant ambient temperature have been considered. Cheng and Minkowycz (Cheng and Minkowycz 1977) studied the problem of natural convection from a vertical flat plate embedded in a saturated porous medium where the wall temperature is a power function of the heat of the plate. The non-

Darcy natural convection from vertical isothermal surfaces in saturated porous media was examined by Plump and Huenefeld (Plump and Huenefeld 1981). They showed that the relative importance of inertia effects in characterized by the modified Grashof number. Lai and Kulacki used both of Darcy and non-Darcy models to study mixed convection from horizontal (Lai and Kulacki 1987) and vertical (Lai and Kulacki 1991) surfaces embedded in saturated porous media. Similar solutions obtained showed that inertia term decreases the rate of heat transfer. Nonsimilarity solutions (Hsieh *et al*. 1993) solved the Darcy model problem of mixed convection from a vertical flat plate in a porous medium under variable surface heat flux and variable wall temperature conditions for entire regime of convection by employing two different transformations. One transformation is for forced convection and the other one is for natural convection. The solution for the entire mixed convection regime was constructed by using the solutions from both systems to avoid the singularity problem at the limiting ends. Also, Nonsimilarity solutions (Hsieh *et al*. 1993) solved the same problem but used a single nonsimilarity parameter, which covers the entire regime of mixed convection. Nakayama and Pop (Nakayama and Pop 1991) proposed a unified similarity transformation to cover all possible similarity solutions for free, forced and mixed convection within Darcy and non-Darcy porous media. The cases they considered were restricted to local similarity approximations. Yu, Lin, and Lu (Yu *et al*. 1991) obtained universal similarity equations of non-Darcy convection along an isothermal vertical plate and uniform heat flux plate in porous media. Gorla and Sidawi (Gorla and Sidawi 1994), Abo-Eldahab and El Aziz (Abo-Eldahab and El Aziz 2005) considered the present work over isothermal stretching surface embedded in a non-Darcian porous medium in the presence of heat generation or absorption. EL-Kabeir and Rashad (EL-Kabeir and Rashad 2006) investigated the influence of variable permeability on free convection flow over a vertical flat plate embedded in a fluid saturated porous medium in the presence of heat sources or sinks. EL-Hakiem and Rashad (EL-Hakiem and Rashad 2007) studied the radiation effect on non-Darcy free convection flow of an incompressible steady viscous fluid adjacent to an isothermal vertical cylinder embedded in a saturated porous medium.

On other hand, the problem of finding similarity solutions of a system of partial differential equations (PDEs) is important in mathematical physics. One of the most powerful methods in order to determine particular solutions to a system of PDEs is based upon the study of their invariance with respect to a one-parameter Lie group of point transformations (Ovsiannikov 1982; Olver 1986; Bluman and Kumei 1989; Ibragimov 1999). If a system of PDEs is invariant under a Lie group of point transformations, we can find special solutions, called invariant solutions, which are invariant under some group of the full group admitted by the system. Especially for a system of non-linear PDEs, the above method provides a systematic and unified procedure in search of invariant solutions. However, many systems of non-linear PDEs in mathematical physics admit only trivial symmetry groups of point transformation, for example, translational symmetries with respect to time or spaces, rotational symmetries and scaling symmetries and so on. Invariant solutions under trivial symmetry groups are constructed by other method or assumption based on physical considerations. Also, in physics, solutions describing physical phenomenon and the well known solutions are discussed and determining theirs ODEs are one of the most important problem in finding similarity solution. Yurusoy and Pakdemirli (Yurusoy and Pakdemirli 1997) found Symmetry reductions of unsteady three-dimensional boundary layers of some non-Newtonian fluids. They (Yurusoy and Pakdemirli 1999) have obtained exact solutions of boundary layer equations of a special non-Newtionian fluid over a stretching sheet. Kalpakides and Balassas (Kalpakides and Balassas 2004) studied the free convective boundary layer problem of an electrically conducting fluid over an elastic surface using group theoretic method. Similarity reductions for problem of radiative and magnetic field effects on free convection and mass-transfer flow past a semi-infinite flat plate were investigated by (Ibrahim *et al*. 2005; Sivasankaran *et al*. 2006; Sivasankaran *et al*. 2006) studied coupled heat and mass transfer fluid flow by natural convection past an inclined semi-infinite porous surface using Lie group analysis. EL-Hakiem *et al*. (EL-Hakiem *et al*.) presented group theoretic analysis of unsteady free convection flow over a continuous moving vertical plate embedded in a fluid-saturated porous medium in the presence of magnetic field effect.

The main purpose of this paper is to show that the use of the Lie groups of transformations in the expanded space of variables including the governing equations parameters that comprise the balance laws of mass, linear momentum, and energy of the problem enables one to enrich the concept of similarity reductions as applied to PDEs. In addition, by determining the transformation group under which a given partial differential equations is invariant, we can obtain information about the invariants and symmetries of those equations, which can be used to determine the similarity variables that will reduce the number of independent variables in the system. Finally, we have obtained similarity reductions of governing nonlinear equations which agreement with previously works were studied in references (Gorla and Sidawi 1994; Abo-Eldahab and El Aziz 2005) .A parametric study of the physical parameters is conducted and a representative set of numerical results for the velocity and temperature, profiles as well as the local skin-friction coefficient, an the rate of heat transfer is illustrated graphically to show interesting features of the solutions.

2 MATHEMATICAL FORMULATIONS

Consider heat transfer and hydromagnetic flow by free convection inclined a semi-infinite porous vertical plate stretching in the x-direction with a velocity bx embedded in a non-Darican porous medium in the presence of thermal radiation effects. The y-direction makes an angle Ω with the horizontal line while the z-direction is normal to the plate surface. A uniform magnetic field of strength B_0 is applied in the z-direction which produces magnetic effects in both the x and y directions. This is done in this way so as to allow suppression of convective flow in these directions. The surface of the plate is maintained at a uniform heat flux q_w. Temperature of the fluid at the surface T_w is assumed to be constant and is higher than the corresponding values T_∞ faraway from the surface unless stated otherwise. In addition, constant fluid suction or injection is imposed at the surface the plate in the z-direction. The Flow model and physical coordinate of the problem is shown in Figure 1. The fluid is assumed to be Newtonian, viscous, electrically conducting, gray, and absorbing/emitting radiation, but non-scattering. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. The surface of the plate is assumed to be electrically insulating. All fluid properties are assumed constant except the density in the buoyancy terms of the x- and y-momentum equations. The viscous dissipation, the external electric field, the induced magnetic field the edge effects are assumed to be negligible. The thermophysical properties of the fluid and porous media are constant. All dependent variables will be independent of the y-direction (Gorla and Sidawi 1994; Abo-Eldahab and El Aziz 2005).

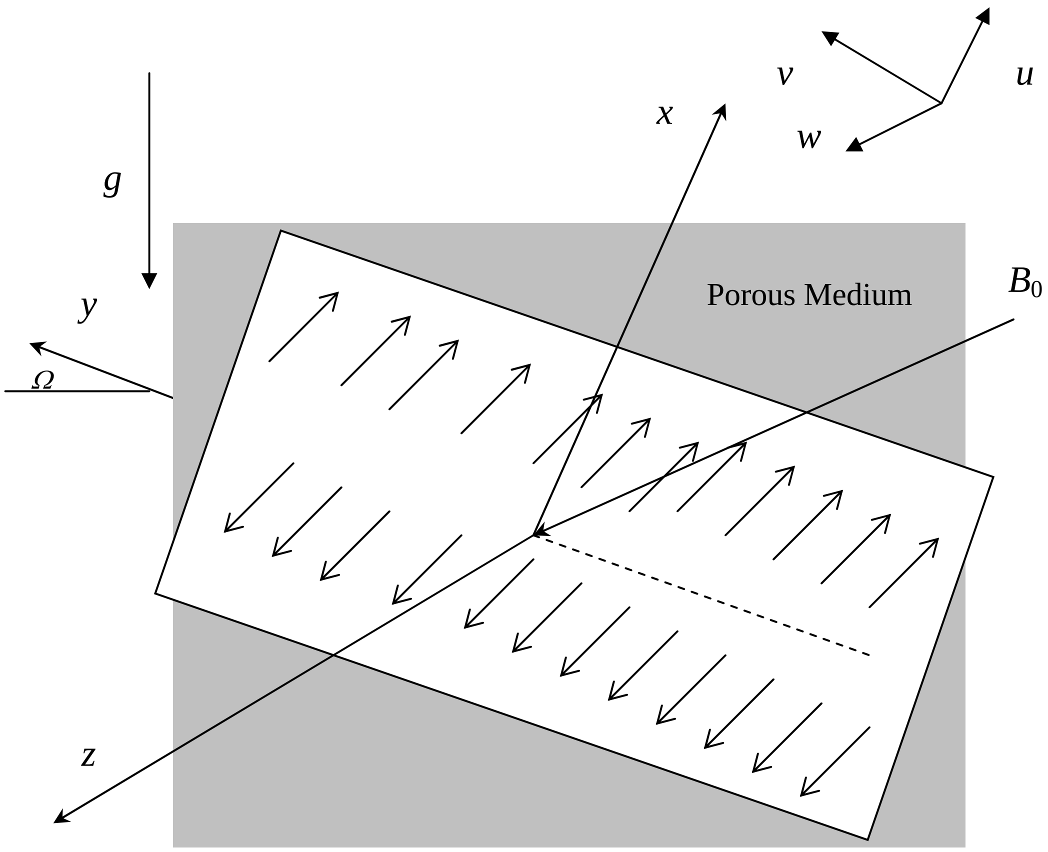

Figure 1: Flow model and physical co-ordinate system

Taking all of the above assumptions into consideration and invoking the boundary-layer and Boussinesq approximations, the governing equations including the porous medium the non-Darcian inertia and radiation effects, can be written in dimensional form as:

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=\upsilon\frac{\partial^2 u}{\partial z^2}+g\beta(T-T_\infty)\cos\Omega-\frac{\sigma B_0^2}{\rho}u-\frac{\upsilon}{K}u-Cu^2 \tag{2}$$

$$w\frac{\partial v}{\partial z}=\upsilon\frac{\partial^2 v}{\partial z^2}+g\beta(T-T_\infty)\sin\Omega-\frac{\sigma B_0^2}{\rho}v-\frac{\upsilon}{K}v-Cv^2 \tag{3}$$

$$w\frac{\partial T}{\partial z}=\frac{\upsilon}{\Pr}\frac{\partial^2 T}{\partial z^2}-\frac{1}{\rho C_p}\frac{\partial q^r}{\partial z} \tag{4}$$

where u, v, and w, are the velocity components associated with the directions of increase of the coordinates x, y, and z , respectively. are the fluid velocity, pressure and respectively. T, ρ, υ, C_p, and $\Pr=\rho C_p \upsilon / k$ are the temperature of fluid, density, kinematic viscosity, specific heat at constant pressure, and the effective Prandtl number, respectively. σ, B_0, and q^r are the fluid electrical conductivity, magnetic induction, and radiative heat flux, respectively. k, g, β, T_∞, and Ω are the thermal conductivity, the acceleration due to gravity, coefficient of thermal expansion, free stream temperature, and the inclination angle, respectively.

The boundary conditions for this problem are given by

$$u(x,0) = bx\,,\ v(x,0) = 0\,,\ w(x,0) = w_0\ \ \theta_z(x,0) = -\theta_w(x,0) \tag{5}$$

$$u(x,\infty) = 0\,,\ v(x,\infty) = 0\,,\ w_z(x,0) = 0\,,\ \theta(x,\infty) = 0 \tag{6}$$

where $\theta = T - T_\infty$, Also $\theta_w = q_w / k$ is a prescribed function along the boundary surface z =0. Also, w_0 is the suction (<0) or injection (>0) velocity and k is the fluid thermal conductivity.

In addition, the radiative heat flux q^r is described according to the Rosseland approximation such that:

$$\frac{\partial q^r}{\partial z} = -\frac{4\sigma_1}{3\chi}\frac{\partial T^4}{\partial z} \tag{7}$$

where σ_1 and χ are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. As done by Raptis (Raptis 1998), the fluid-phase temperature differences within the flow are assumed to be sufficiently small so that T^4 may be expressed as a linear function of temperature. This is done by expanding T^4 in a Taylor series about the free-stream temperature T_∞ and neglecting higher-order terms to yield

$$T^4 = 4T_\infty^4 T - 3T_\infty^4 \tag{8}$$

By using equations (7) and (8) in the last term of equation (4), we obtain

$$\frac{\partial q^r}{\partial z} = -\frac{16\sigma_1 T_\infty^3}{3\chi}\frac{\partial^2 T}{\partial z^2} \tag{9}$$

3 LIE SYMMETRY GROUP METHOD

In this section, we briefly discuss how to determine Lie point symmetry generators admitted by equations (1) to (4). We use these generators to successively reduce equations (1) to (4) to ordinary differential equation which can be solved numerically. Consider the one-parameter Lie group of infinitesimal transformations in (x,z,u,v,w,θ) given by:

$$\begin{aligned}
x^* &= x + \varepsilon\xi^1(x,z,u,v,w,\theta) + O(\varepsilon^2)\\
z^* &= z + \varepsilon\xi^2(x,z,u,v,w,\theta) + O(\varepsilon^2)\\
u^* &= u + \varepsilon\mu^1(x,z,u,v,w,\theta) + O(\varepsilon^2)\\
v^* &= v + \varepsilon\mu^2(x,z,u,v,w,\theta) + O(\varepsilon^2)
\end{aligned}$$

$$w^* = w + \varepsilon\mu^3(x,z,u,v,w,\theta) + O(\varepsilon^2)$$
$$\theta^* = \theta + \varepsilon\mu^4(x,z,u,v,w,\theta) + O(\varepsilon^2) \qquad (10)$$

where ε is the Lie group parameter. A system of partial differential equations (1)-(4) is said to admit a symmetry generated by the vector field

$$X = \xi^1(x,z,u,v,w,\theta)\frac{\partial}{\partial x} + \xi^2(x,z,u,v,w,\theta)\frac{\partial}{\partial z} + \mu^1(x,z,u,v,w,\theta)\frac{\partial}{\partial u} + \mu^2(x,z,u,v,w,\theta)\frac{\partial}{\partial v} + \mu^3(x,z,u,v,w,\theta)\frac{\partial}{\partial w} + \mu^4(x,z,u,v,w,\theta)\frac{\partial}{\partial \theta} \qquad (11)$$

if it is left invariant by the transformation $(x,z,u,v,w,\theta) \to (x^*, z^*, u^*, v^*, w^*, \theta^*)$

Equivalently, we can obtain $(x^*, z^*, u^*, v^*, w^*, \theta^*)$ by solving

$$\frac{\mathrm{d}x^*}{\mathrm{d}\varepsilon} = \xi^1(x^*, z^*, u^*, v^*, w^*, \theta^*),\ \frac{\mathrm{d}z^*}{\mathrm{d}\varepsilon} = \xi^2(x^*, z^*, u^*, v^*, w^*, \theta^*)$$
$$\frac{\mathrm{d}u^*}{\mathrm{d}\varepsilon} = \mu^1(x^*, z^*, u^*, v^*, w^*, \theta^*),\ \frac{\mathrm{d}v^*}{\mathrm{d}\varepsilon} = \mu^2(x^*, z^*, u^*, v^*, w^*, \theta^*)$$
$$\frac{\mathrm{d}w^*}{\mathrm{d}\varepsilon} = \mu^3(x^*, z^*, u^*, v^*, w^*, \theta^*),\ \frac{\mathrm{d}\theta^*}{\mathrm{d}\varepsilon} = \mu^4(x^*, z^*, u^*, v^*, w^*, \theta^*) \qquad (12)$$

subjected to the initial conditions

$$(x^*, z^*, u^*, v^*, w^*, \theta^*)\big|_{\varepsilon=0} = (x,z,u,v,w,\theta) \qquad (13)$$

The solutions $\xi^1=\xi^1(x, z, u, v, w,\theta)$, $\xi^2=\xi^2(x, z, u, v, w,\theta)$, $\mu^1=\mu^1(x, z, u, v, w,\theta)$, $\mu^2=\mu^2(x, z, u, v, w,\theta)$, $\mu^3=\mu^3(x, z, u, v, w,\theta)$ and $\mu^4=\mu^4(x, z, u, v, w,\theta)$ are invariant under symmetry equation (11) if

$$\Psi_u = X(u - u(x,z,)) = 0 \text{ when } u = u(x,z)$$
$$\Psi_v = X(v - v(x,z)) = 0 \text{ when } v = v(x,z)$$
$$\Psi_w = X(w - w(x,z)) = 0 \text{ when } w = w(x,z)$$
$$\Psi_\theta = X(\theta - \theta(x,z)) = 0 \text{ when } \theta = \theta(x,z) \qquad (14)$$

These conditions can be expressed by using the characteristic of the group, which are

$$\Psi_u = \mu^1 - \xi^1\frac{\partial u}{\partial x} - \xi^2\frac{\partial u}{\partial z},\quad \Psi_v = \mu^2 - \xi^1\frac{\partial v}{\partial x} - \xi^2\frac{\partial v}{\partial z},\quad \Psi_w = \mu^3 - \xi^1\frac{\partial w}{\partial x} - \xi^2\frac{\partial w}{\partial z},\ \text{and}$$
$$\Psi_\theta = \mu^4 - \xi^1\frac{\partial \theta}{\partial x} - \xi^2\frac{\partial \theta}{\partial z} \qquad (15)$$

From equation (14), the solutions The solutions $\xi^1=\xi^1(x, z, u, v, w,\theta)$, $\xi^2=\xi^2(x, z, u, v, w,\theta)$, $\mu^1=\mu^1(x, z, u, v, w,\theta)$, $\mu^2=\mu^2(x, z, u, v, w,\theta)$, $\mu^3=\mu^3(x, z, u, v, w,\theta)$ and $\mu^4=\mu^4(x, z, u, v, w,\theta)$ are invariant provided that

$$\Psi_u = 0 \text{ when } u = u(x,z),\ \Psi_v = 0 \text{ when } v = v(x,z),$$
$$\Psi_w = 0 \text{ when } w = w(x,z),\ \Psi_\theta = 0 \text{ when } \theta = \theta(x,z) \quad (16)$$

Thus, equations of (15) can be rewritten as

$$\mu^1 = \xi^1 \frac{\partial u}{\partial x} + \xi^2 \frac{\partial u}{\partial z},\ \mu^2 = \xi^1 \frac{\partial v}{\partial x} + \xi^2 \frac{\partial v}{\partial z},\ \mu^3 = \xi^1 \frac{\partial w}{\partial x} + \xi^2 \frac{\partial w}{\partial z}$$
and $$\mu^4 = \xi^1 \frac{\partial \theta}{\partial x} + \xi^2 \frac{\partial \theta}{\partial z} \quad (17)$$

Equation (17) is called the invariant surface conditions, which are quasi-linear equations. The subsidiary equations may be expressed as:

$$\frac{\mathrm{d}x}{\xi^1(x,z,u,v,w,\theta)} = \frac{\mathrm{d}z}{\xi^2(x,z,u,v,w,\theta)} = \frac{\mathrm{d}u}{\mu^1(x,z,u,v,w,\theta)} = \frac{\mathrm{d}v}{\mu^2(x,z,u,v,w,\theta)}$$
$$= \frac{\mathrm{d}w}{\mu^3(x,z,u,v,w,\theta)} = \frac{\mathrm{d}\theta}{\mu^4(x,z,u,v,w,\theta)} \quad (18)$$

A vector X given by (10) is said to be a Lie point symmetry vector field for equations (1) to (4) if

$$X^{(1)}(u_x + w_z) = 0 \quad (19)$$

$$X^{(2)}\left(uu_x + wu_z - \upsilon u_{zz} - g\beta\theta\cos\Omega + \left(\rho^{-1}\sigma B_0^2 + K^{-1}\upsilon\right)u + Cu^2\right) = 0 \quad (20)$$

$$X^{(2)}\left(wv_z - \upsilon v_{zz} - g\beta\theta\sin\Omega + \left(\rho^{-1}\sigma B_0^2 + K^{-1}\upsilon\right)v + Cv^2\right) = 0 \quad (21)$$

$$X^{(2)}\left(wT_z - \Pr^{-1}\upsilon\left(1 + (4R_d/3)\right)\theta_{zz}\right) = 0 \quad (22)$$

where $X^{(2)}$ the second prolongation of X is given by

$$X^{(2)} = X + \mu_x^1 \frac{\partial}{\partial u_x} + \mu_z^1 \frac{\partial}{\partial u_z} \mu_z^2 \frac{\partial}{\partial v_z} + \mu_z^3 \frac{\partial}{\partial w_x} + \mu_z^4 \frac{\partial}{\partial u_z} + \mu_{zz}^1 \frac{\partial}{\partial u_{zz}} + \mu_{zz}^2 \frac{\partial}{\partial v_{zz}} + \mu_{zz}^4 \frac{\partial}{\partial \theta_{zz}} \quad (23)$$

where, the components $\mu_x^1, \mu_z^1, \mu_z^2, \mu_z^3, \mu_z^4, \mu_{zz}^1, \mu_{zz}^2, \mu_{zz}^4$ can be determined from the following expressions:

$$\mu_x^1 = D_x(\mu^1) - u_x D_x(\xi^1) - u_z D_x(\xi^2),\ \mu_z^1 = D_z(\mu^1) - u_x D_z(\xi^1) - u_z D_z(\xi^2)$$
$$\mu_z^2 = D_z(\mu^2) - v_x D_z(\xi^1) - v_z D_z(\xi^2),\ \mu_z^3 = D_z(\mu^3) - w_x D_z(\xi^1) - w_z D_z(\xi^2)$$
$$\mu_z^4 = D_z(\mu^4) - \theta_x D_z(\xi^1) - \theta_z D_z(\xi^2),\ \mu_{zz}^1 = D_z(\mu_z^1) - u_{xz} D_z(\xi^1) - u_{zz} D_z(\xi^2)$$
$$\mu_{zz}^2 = D_z(\mu_z^2) - v_{xz} D_z(\xi^1) - v_{zz} D_z(\xi^2),\ \mu_{zz}^4 = D_z(\mu_z^4) - \theta_{xz} D_z(\xi^1) - \theta_{zz} D_z(\xi^2) \tag{24}$$

and D_x and D_z are the operators of total differentiation with respect to x and z, respectively. By carrying out a straightforward and tedious algebra for equations (19) to (22) using equation (23), we finally obtain the form of the so-called infinitesimals,

$$\xi^1 = C_1 x,\ \xi^2 = C_2 x + C_3 z,\ \mu^1 = (C_1 - 2C_3)u + h_1(x,z),\ \mu^2 = 2C_3 v + h_2(x,z),$$
$$\mu^3 = C_2 u - C_3 w + h_3(x,z),\ \mu^4 = 2C_3\theta + h_4(x,z) \tag{25}$$

where C_1, C_2 and C_3 are arbitrary constants and h_1, h_2, h_3, and h_4 are an arbitrary functions depends on x, z. Hence, the system of nonlinear equations (1) to (4) has the seven-parameter Lie group of point symmetries generated by

$$X_1 = x\frac{\partial}{\partial x} + u\frac{\partial}{\partial u},\ X_2 = x\frac{\partial}{\partial z} + u\frac{\partial}{\partial w},\ X_3 = z\frac{\partial}{\partial z} - 2u\frac{\partial}{\partial u} + 2v\frac{\partial}{\partial v} - w\frac{\partial}{\partial w} + 2\theta\frac{\partial}{\partial \theta},$$
$$X_4 = h_1(x,z)\frac{\partial}{\partial u},\ X_5 = h_2(x,z)\frac{\partial}{\partial v},\ X_6 = h_3(x,z)\frac{\partial}{\partial w},\ X_7 = h_4(x,z)\frac{\partial}{\partial \theta} \tag{26}$$

and the corresponding invariant transformations are

$$(x,z,u,v,w,\theta) \to (e^{\varepsilon}x, z, e^{\varepsilon}u, v, w, \theta)$$
$$(x,z,u,v,w,\theta) \to (x, z+\varepsilon x, u, v, w+\varepsilon u, \theta)$$
$$(x,z,u,v,w,\theta) \to (x, e^{\varepsilon}z, e^{-2\varepsilon}u, e^{2\varepsilon}v, e^{-\varepsilon}w, e^{2\varepsilon}\theta)$$
$$(x,z,u,v,w,\theta) \to (x, z, u+\varepsilon h_1, v, w, \theta)$$
$$(x,z,u,v,w,\theta) \to (x, z, u, v+\varepsilon h_2, w, \theta)$$
$$(x,z,u,v,w,\theta) \to (x, z, u, v, w+\varepsilon h_3, \theta)$$
$$(x,z,u,v,w,\theta) \to (x, z, u, v, w, \theta+\varepsilon h_4) \tag{27}$$

4. REDUCTION TO ORDINARY DIFFERENTIAL EQUATIONS

In this section, we will produce the similarity transformations and solutions using the infinitesimals given in equation (25). We first transform the equations into an ordinary differential system, and solve this system numerically using a Runge-Kutta method with shooting technique. Leaving the details of the procedure to references (Olver 1986; Bluman and Kumei 1989), we choose only the scaling transformation (i.e. $C_1 = 1$, $C_2 = C_3 = 0$ and $h_1 = h_2 = h_3 = h_4 = 0$).The equations for similarity transformations are

$$\frac{dx}{x}=\frac{dz}{0}=\frac{du}{u}=\frac{dv}{0}=\frac{dw}{0}=\frac{d\theta}{0} \tag{28}$$

Solving equation (28), we have the following functions and similarity variable

$$u=k_1xF'(\eta)+k_1^*M(\eta),\ v=k_2N(\eta),\ w=-k_3F(\eta),\ \theta=k_4H(\eta),\ \ \eta=k_5z \tag{29}$$

where k_1, k_2, k_3, k_4 and k_5 are an arbitrary constants will be determined later. Consequently, substituting equation (29) into the original system of motion and energy equations (1) to (4), and boundary conditions equations (5) to (6), (taking equation (9) into account), we obtain the following similarity equations and boundary conditions:

$$F'''+FF''-F'^2-2\Gamma F'M\cos\Omega-\left(Ha^2+Da^{-1}\right)F'=0 \tag{30}$$

$$M''+FM'-MF'-\Gamma M^2\cos\Omega-\left(Ha^2+Da^{-1}\right)M+H=0 \tag{31}$$

$$N''+FN'-\Gamma N^2\sin\Omega-\left(Ha^2+Da^{-1}\right)N+H=0 \tag{32}$$

$$\frac{1}{\Pr}\left(1+\frac{4R_d}{3}\right)H''+FH'=0 \tag{33}$$

$$\begin{gathered}F(0)=F_w,\ \ F'(0)=1,\ F'(\infty)=0,\ M(0)=0,\ M(\infty)=0,\\ N(0)=0, N(\infty)=0,\ \theta'(0)=-1,\ \theta(\infty)=0\end{gathered} \tag{34}$$

To avoid the fluid properties appearing explicitly in the coefficients of the above equations, we have introduced four new appropriate arbitrary constants as follows:

$$\begin{gathered}k_1=b,\ \ k_1^*=(g\beta\theta_w\cos\Omega\sqrt{\upsilon}/b^{3/2}),\ k_2=(g\beta\theta_w\sin\Omega\sqrt{\upsilon}/b^{3/2}),\ k_3=(\sqrt{b\upsilon}),\ k_4=\theta_w\sqrt{b/\upsilon},\\ k_5=\sqrt{b/\upsilon}\end{gathered} \tag{35}$$

Also, a prime denotes partial differentiation with respect to η and $Ha^2=\sigma B_0^2/\rho b$, $Da^{-1}=\upsilon/bK$, $\Gamma=Cg\beta\theta_w\sqrt{\upsilon}/b^{3/2}$ are the square of the magnetic Hartmann number, the inverse Darcy number, and the dimensionless porous medium inertia coefficient, respectively. $Q=Q_0/\rho C_p b$, $F_w=-w_0/\sqrt{b\upsilon}$ are the dimensionless heat generation/absorption coefficient, and wall mass transfer coefficient, respectively. It is should be noted that positive values of F_w indicate fluid suction at the plate surface while negative values of F_w indicate fluid blowing or injection at the wall.

Important physical parameters for this flow and heat transfer situation are the skin-friction coefficients in the x and y directions and the local Nusselt number. The shear stresses at the stretching surface are given by

$$\tau_{zx} = \mu \frac{\partial u}{\partial z}(x,0)$$
$$= \frac{\mu}{\sqrt{\upsilon}}\left(bxF''(0) + (g\beta\theta_w \cos\Omega\sqrt{\upsilon}/b^{3/2})M'(0)\right) \tag{36}$$

$$\tau_{zy} = \mu \frac{\partial v}{\partial z}(x,0) = (\mu g\beta\theta_w \sin\Omega / b)N'(0) \tag{37}$$

where $\mu = (\rho\upsilon)$ is the dynamic viscosity of the fluid. Upon quantities of τ_{zx} and τ_{zy} by $\mu = \rho(bx)^2/2$, the following respective expressions for the skin-friction coefficients in the x and y directions result:

$$C_{fx} = \frac{2}{\mathrm{Re}_x}\left(\frac{Gr_x}{\mathrm{Re}_x^2}\cos\Omega M'(0) - 1\right) \tag{38}$$

$$C_{fy} = \frac{2Gr_x}{\mathrm{Re}_x^3}\sin\Omega N'(0) \tag{39}$$

where $Gr_x = g\beta\theta_w x^3/\upsilon^2$ and $\mathrm{Re}_x = bx^2/\upsilon$ are the local Grashof and Reynolds number, respectively.

The local Nusselt number for this situation can then be defined as

$$Nu_L = \frac{hx}{k} = \frac{q_w x}{k\theta} = \mathrm{Re}_L^{-1/2}\ \frac{1}{H(0)} \tag{40}$$

where h is the local heat transfer coefficient, L is a characteristic plate length, and $\mathrm{Re}_L = bL^2/\upsilon$ is the Reynolds number at $x = L$.

5 RESULTS AND DISCUSSION

The transformation of the governing partial differential equations into ordinary ones using the above similarity variables are solved numerically using the Runge–Kutta integration scheme with shooting method. A representative set of graphical results for water (Pr=6.7) flow saturated porous medium ($Da = 100$) along an impermeable surface ($F_w = 0$) is maintained at constant heat flux with slight thermal radiation effect ($R_d = 0.2$), including inertia resistance ($\Gamma = 2.0$) and in the presence of a magnetic field ($Ha = 1.5$), where the *y*-direction makes an angle ($\Omega = 45$) with a horizontal line as a reference is presented in Figures 2 to14 to illustrate the physical influence of the magnetic Hartmann number *Ha*, the dimensionless inertia parameter Γ, Dacry number *Da*, the wall mass transfer coefficient F_W, the Prandtl number *Pr*, and radiation parameter R_d on the variables of the fluid's *x*-component of velocity (F' and *M*), *y*-component of the velocity (*N*), *z*-component of the velocity (*F*)and temperature of the fluid (H). From Figures 2 to 14. It is found that increasing inertia parameter Γ, Hartmann number Ha, or decreasing the Darcy number Da decreases both parts of (F' and M) as well as, y-component of the velocity (N) and z-component of the velocity (F) to decrease. That is because the application of a magnetic field in the z-direction to an electrically-conducting fluid gives rise to a flow resistive force called the Lorentz force. This force will have components in both the x- and y-directions, and also both the permeability of porous media and resistance to the flow. This is due to the fact that Γ, Ha and Da^{-1} represent additional resistance to flow, thus, slowing the fluid flow and enhanced temperature of the fluid (H), and hence reduced the wall heat transfer and both of the skin-friction coefficients in the x-, y-directions, which are illustrated in table 1. On other hand, it is obvious that, as the wall mass transfer coefficient F_W, Prandtl number Pr, increases or decreasing radiation parameter R_d, both the fluid hydrodynamic and thermal boundary layers decrease. Thus, all the fluid's velocity components (except the z-component) as well as its temperature decrease at every point above the plate.

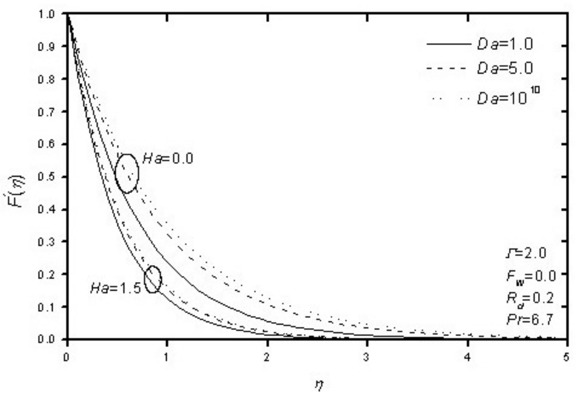

Figure 2: Effects of Hartmann number Ha and Darcy number Da on the fluid velocity of x-direction (F').

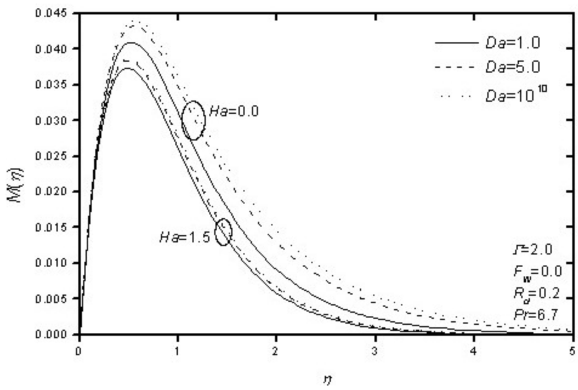

Figure 3: Effects of Hartmann number Ha and Darcy number Da on the fluid velocity of x-direction (M).

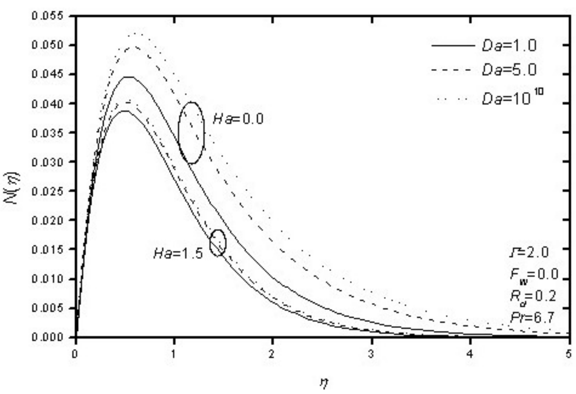

Figure 4: Effects of Hartmann number Ha and Darcy number Da on the fluid velocity of y-direction (N).

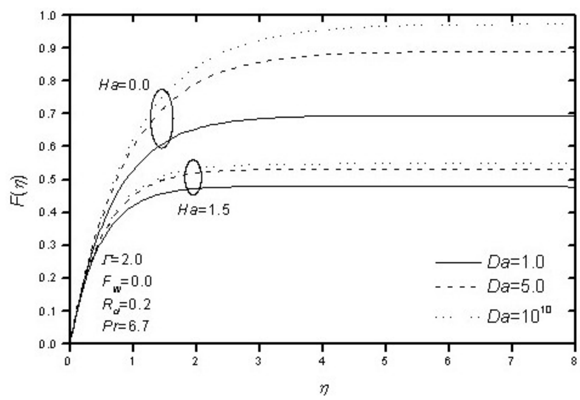

Figure 5: Effects of Hartmann number Ha and Darcy number Da on the fluid velocity of z-direction (F).

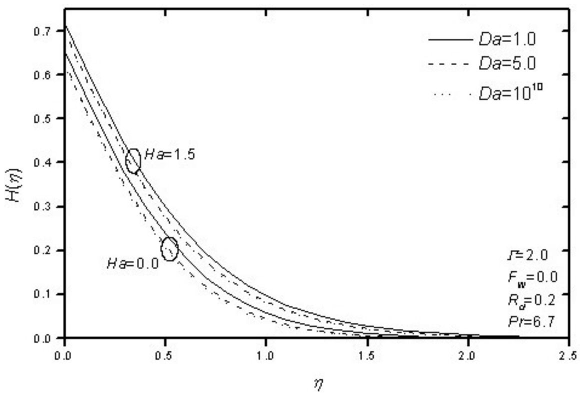

Figure 6: Effects of Hartmann number *Ha* and Darcy number *Da* on the fluid temperature (*H*).

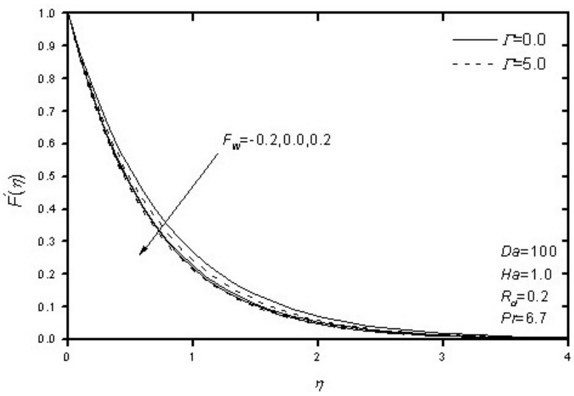

Figure 7: Effects of inertia parameter Γ and blowing/suction parameter F_W on the fluid velocity of *x*-direction (F').

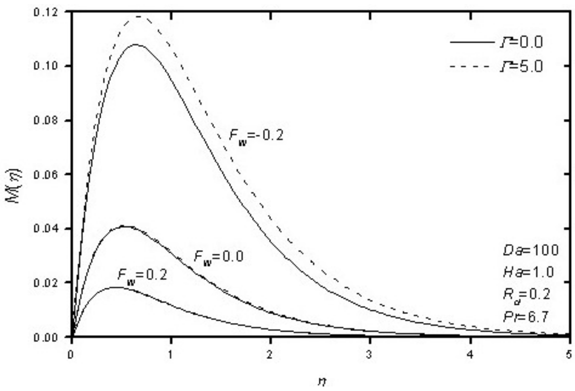

Figure 8: Effects of inertia parameter Γ and blowing/suction parameter F_W on the fluid velocity of *x*-direction (*M*).

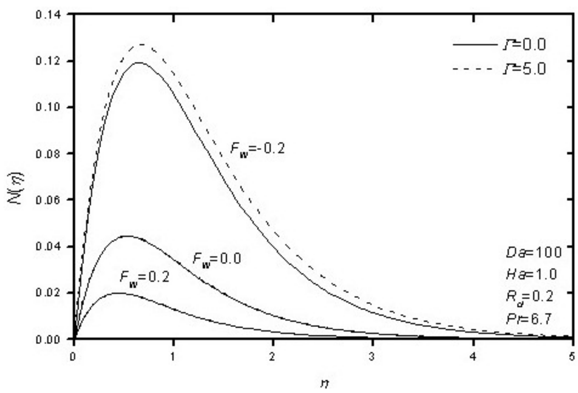

Figure 9: Effects of inertia parameter Γ and blowing/suction parameter F_W on the fluid velocity of *y*-direction (*N*).

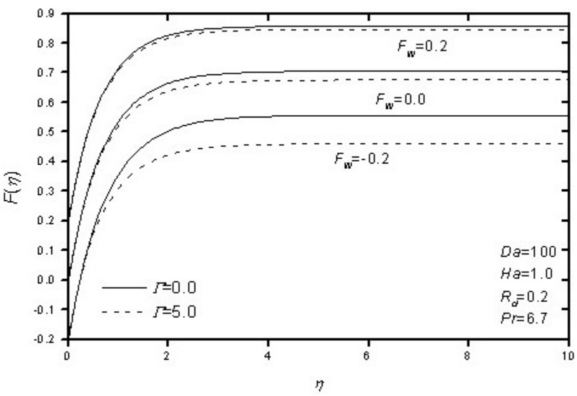

Figure 10: Effects of inertia parameter Γ and blowing/suction parameter F_W on the fluid velocity of *z*-direction (*F*).

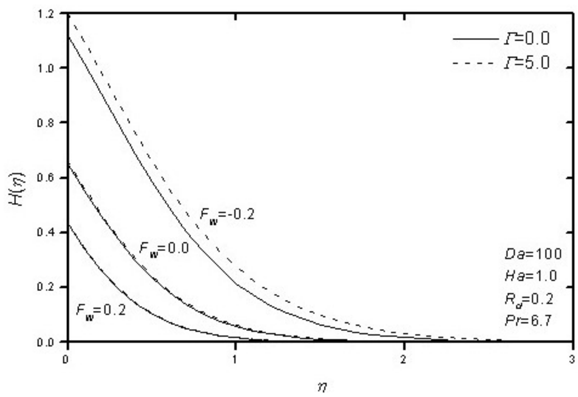

Figure 11: Effects of inertia parameter Γ and blowing/suction parameter F_W on the fluid temperature (*H*).

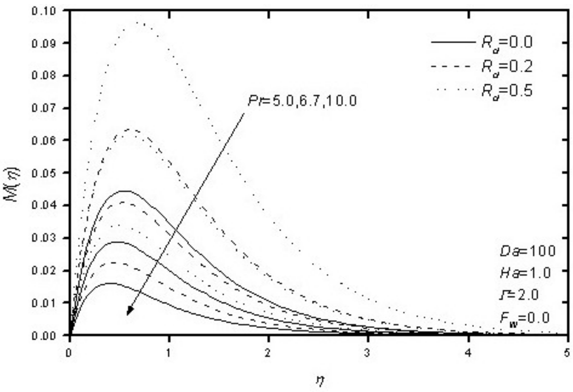

Figure 12: Effects of Prandtl number *Pr* and radiation parameter R_d on the fluid velocity of *x*-direction (*M*).

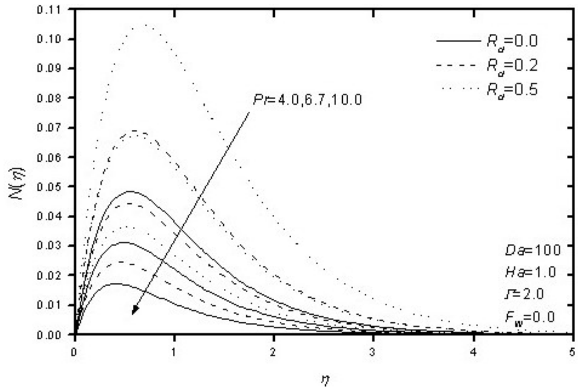

Figure 13: Effects of Prandtl number *Pr* and radiation parameter R_d on the fluid velocity of *y*-direction (*N*).

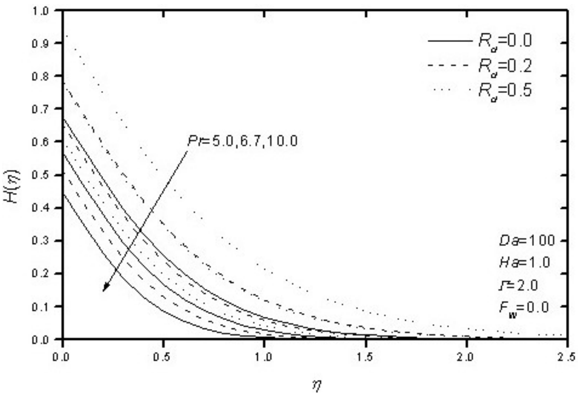

Figure 14: Effects of Prandtl number *Pr* and radiation parameter R_d on the fluid temperature (*H*).

Moreover, form Table 1, the wall heat transfer enhances and both of the skin-friction coefficients in the *x, y* directions, reduces with an increasing the values of F_W, Pr or decreasing the values of R_d. Obviously, changes in the values of *Pr*, R_d will cause no changes in the profiles of the fluid's *x*-component of velocity (F') and *z*-component of the velocity (*F*). For this reason, no figures for these variables are presented herein and no effects of the skin-friction coefficient in the *x*-direction $F''(0)$. That is because Equations (29) and (34) uncoupled from the equations.

6 CONCLUDING REMARKS

Lie symmetry group method is applicable to the problem of steady, laminar free convection from a vertical stretching surface saturated porous medium in the presence of magnetic field, inertia force, thermal radiation and wall suction/injection effects with uniform heat flux. By using Lie symmetry technique to non-linear partial differential equations, which leads to similarity variables that may be used to reduce the number of independent variables in partial differential equations. By determining the transformation group under which a given partial differential equation is invariant, we can obtain information about the invariants and symmetries of those equations. This information can be used to determine the similarity variables that will reduce the number of independent variables in the system. Finally, we have obtained similarity reductions of nonlinear equations of motion equations (1) to (4) which agreement with works were considered in references (Gorla and Sidawi 1994; Abo-Eldahab and El Aziz 2005). We solved numerically the resulting differential equations with its boundary conditions using shooting method with Runge-Kutta scheme. We have discussed and plotted the effects of the Prandtl number, Hartmann number, Darcy number, porous-medium inertia coefficient and radiation parameter, on velocity and temperature profiles as well as skin friction coefficients and heat transfer.

Table 1: Values of both of the skin-friction coefficient in the x-,y-directions and the wall heat transfer, respectively ($F''(0)$, $M'(0)$, $N'(0)$ and $1/H(0)$)for various values of Ha, Da^{-1}, Γ, F_W, R_d and Pr.

Ha	Da	Γ	F_W	R_d	Pr	$F''(0)$	$M'(0)$	$N'(0)$	$1/H(0)$
0.0	1.0	2.0	0.0	0.2	6.7	-1.4400	0.1999	0.2108	1.52149
0.0	5.0	2.0	0.0	0.2	6.7	-1.1309	0.2013	0.2182	1.58918
0.0	10^{10}	2.0	0.0	0.2	6.7	-1.0393	0.2018	0.2219	1.60928
1.5	1.0	2.0	0.0	0.2	6.7	-2.0767	0.1976	0.2035	1.38420
1.5	5.0	2.0	0.0	0.2	6.7	-1.8751	0.1983	0.2052	1.42716
1.5	10^{10}	2.0	0.0	0.2	6.7	-1.8212	0.1985	0.2058	1.43873
1.0	100	0.0	-0.2	0.2	6.7	-1.3213	0.4170	0.4455	0.88946
1.0	100	0.0	0.0	0.2	6.7	-1.4177	0.1990	0.2101	1.52759
1.0	100	0.0	0.2	0.2	6.7	-1.5213	0.1102	0.1152	2.28604
1.0	100	5.0	-0.2	0.2	6.7	-1.4827	0.4495	0.4739	0.83180
1.0	100	5.0	0.0	0.2	6.7	-1.4814	0.2012	0.2116	1.51054
1.0	100	5.0	0.2	0.2	6.7	-1.5511	0.1104	0.1153	2.28009
1.0	100	2.0	0.0	0.0	5.0	-1.4361	0.2121	0.2238	1.46915
1.0	100	2.0	0.0	0.2	5.0	-1.4363	0.2727	0.2889	1.26697
1.0	100	2.0	0.0	0.5	5.0	-1.4338	0.3668	0.3899	1.05868
1.0	100	2.0	0.0	0.0	6.7	-1.4332	0.1560	0.1638	1.75336
1.0	100	2.0	0.0	0.2	6.7	-1.4335	0.1999	0.2107	1.52074
1.0	100	2.0	0.0	0.5	6.7	-1.4354	0.2675	0.2832	1.28178
1.0	100	2.0	0.0	0.0	10	-1.4353	0.1031	0.1075	2.21415
1.0	100	2.0	0.0	0.2	10	-1.4354	0.1316	0.1377	1.93129
1.0	100	2.0	0.0	0.5	10	-1.4353	0.1751	0.1842	1.64145

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